Reshaping our World
A new world map on an irregular heptahedron

Introduction
Creating world maps by projecting onto a polyhedron is not a new concept. It has been a popular method for controlling distortion in map projections (Snyder, 1992). However, previous polyhedral projections have limited the placement of the projective centers to a symmetric pattern around the Earth. This may not be the best solution since the Earth’s land masses are not evenly distributed around the globe.

The polyhedra that have been used for this purpose in other projections are polyhedron with regular faces. Common polyhedra with regular faces that have been used in projections include the Patroclus solid, which has two square faces and five triangular faces. Regular polyhedra provide more control over the arrangement of the projective centers, enabling further reduction in distortion. The projection shown here uses a seven-faced, irregular polyhedron to map the seven continents.

Further Work
Creating a seven-faced irregular polyhedron projection demonstrated potential for the method of creating map projections. Polyhedra with more faces experience a smaller increase in distortion. Therefore, to create a projection with less than one generating point could be assigned to South America, the distortion in South America and other continents could be reduced by adding more generating points and by finding an optimal arrangement of these points. Different versions could be constructed to minimize area or shape distortion.

Method
Creating an irregular polyhedron projection involves joining multiple gnomonic projections along common apsidal lines, using projective criteria. A boundary must serve as a perpendicular bisector of the adjacent projective centers, that is, it must cross a line between the two points perpendicular to the boundary. The projection created here has seven faces, with the projective centers on each of the seven continents. An outline of the process below:

Step 1: The first step is to define the points that will serve as the center points of the spherical Voronoi diagram for each face. These points are defined in terms of geographic coordinates, longitude (\(\lambda\)) and latitude (\(\phi\)).

Step 2: The next step is to calculate the coordinates of the vertices for the spherical Voronoi diagram. To do this, we must first convert the geographic coordinates to Cartesian coordinates \(x, y, z\) by

\[
\begin{align*}
x &= \cos(\lambda) \cos(\phi) \\
y &= \sin(\lambda) \\
z &= \cos(\lambda) \sin(\phi)
\end{align*}
\]

Calculating the vertices involves inputting three points, denoted in vector notation, from adjacent spherical polygons into a formula (see bottom diagram). This must be done for all sets of adjacent sides. The Cartesian coordinates of the vertices are then converted back to geographic coordinates.

Step 3: Once all vertices are calculated, they can be connected to form the spherical Voronoi diagram and the necessary boundaries.

Step 4: The final step is to project each spherical polygon gnomonically.

References

Finding the Vertices of a Spherical Voronoi Polygon

A sphere in a spherical Voronoi diagram is intersected by the sphere's center point. To calculate the position of each vertex, we use the following formulas:

\[
\begin{align*}
x &= \cos(\lambda) \cos(\phi) \\
y &= \sin(\lambda) \\
z &= \cos(\lambda) \sin(\phi)
\end{align*}
\]

The Gnomonic Projection

The gnomonic projection can be envisioned as a projection on a cone that rests on the Earth. The point of tangency is the origin of the spherical coordinate system. In the projection presented here, there are seven tangent points, one for each continent. The point of tangency is the only point without any distortion, but distortion quickly increases away from the point of tangency. The points of tangency are equidistant from the edge between them for all points along the edge. One property of the gnomonic projection is that all great circle arcs are shown as straight lines. This property makes it useful for areas with large edges such as Africa or Europe. However, this property does not hold for great circle arcs across multiple points.